

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/02R**

Further Pure Mathematics
PAPER 2R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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2 Find the set of values of x for which

(a) $3x - 8 < 5x + 3$ (1)

(b) $4x^2 - 7x + 1 > 6 - 2x^2$ (4)

(c) **both** $3x - 8 < 5x + 3$ **and** $4x^2 - 7x + 1 > 6 - 2x^2$ (1)

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Question 2 continued

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(Total for Question 2 is 6 marks)

3 Given that $y = e^{3x} \sin 2x$

show that $13y + \frac{d^2y}{dx^2} = 6 \frac{dy}{dx}$

(8)

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Question 3 continued

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(Total for Question 3 is 8 marks)

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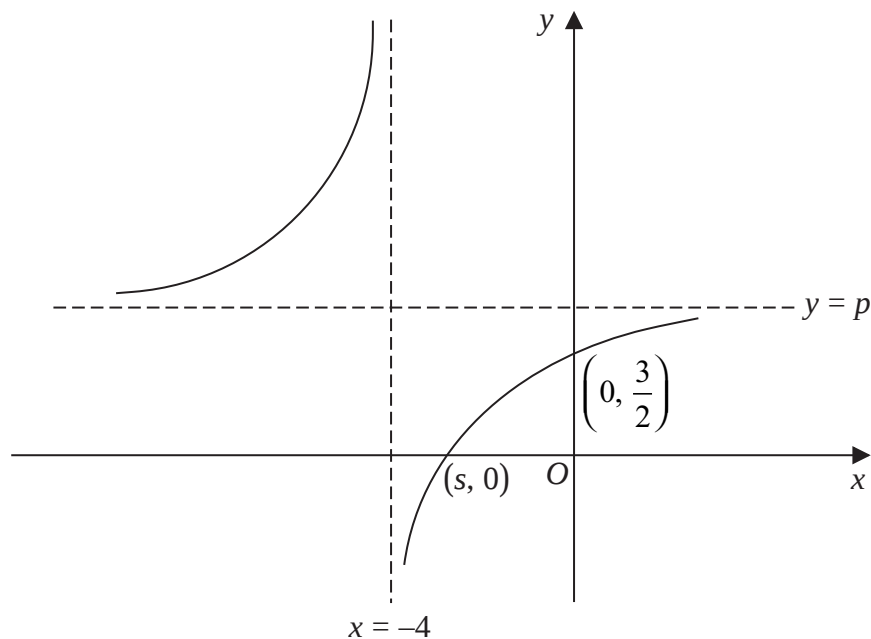


Figure 1

Figure 1 shows part of the curve C with equation

$$y = \frac{2x + q}{x + r} \quad x \neq -r$$

where q and r are integers.

The asymptote to C that is parallel to the y -axis has equation $x = -4$

The asymptote to C that is parallel to the x -axis has equation $y = p$

(a) Write down

(i) the value of p

(ii) the value of r

(2)

Given that C crosses the y -axis at the point with coordinates $\left(0, \frac{3}{2}\right)$

(b) find the value of q

(2)

Given that C crosses the x -axis at the point with coordinates $(s, 0)$

(c) find the value of s

(2)

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Question 4 continued

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(Total for Question 4 is 6 marks)

- 5 The line l with gradient $-\frac{1}{12}$ passes through the points A and B with coordinates $(p, 10)$ and $(123, 0)$ respectively.

(a) Show that $p = 3$

(2)

(b) Find an equation for l in the form $rx + sy + t = 0$ where r, s and t are integers.

(2)

The line k is perpendicular to l and passes through the point A .

(c) Find an equation for k in the form $y = mx + c$

(3)

Line k intersects the x -axis at the point C .

(d) Find the exact area of triangle ABC .

(4)

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Question 5 continued

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Question 5 continued

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Question 5 continued

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(Total for Question 5 is 11 marks)

6

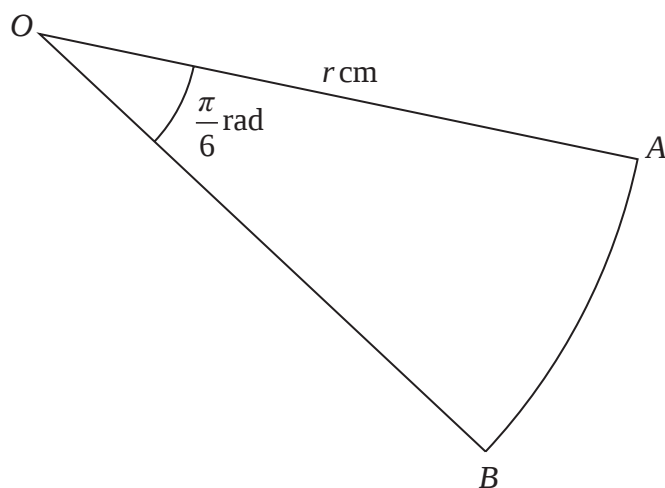
Diagram **NOT**
accurately drawn**Figure 2**

Figure 2 shows the sector OAB of a circle with centre O and radius r cm.

$$\angle AOB = \frac{\pi}{6} \text{ radians} \quad OA = OB = r \text{ cm}$$

The area of the sector is increasing in such a way that the size of $\angle AOB$ remains constant, and the lengths OA and OB are both increasing at a constant rate of 0.2 cm/s

Find the exact rate of change, in cm^2/s , of the area of the sector when the length of

arc AB is $\frac{5\pi}{2}$

(6)

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Question 6 continued

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7

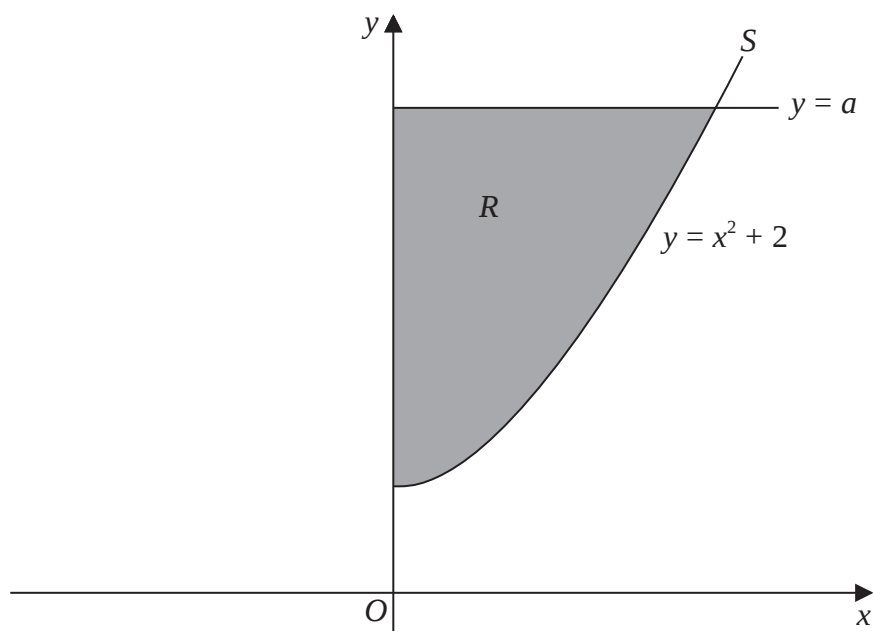


Diagram **NOT**
accurately drawn

Figure 3

Figure 3 shows part of the curve S with equation $y = x^2 + 2$

The finite region R , shown shaded in Figure 3, is bounded by S , the y -axis and the line with equation $y = a$ where $a > 2$

The region R is rotated through 360° about the y -axis to generate a solid with volume 18π

Use algebraic integration to find the value of a

(8)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 8 marks)

- 8 The quadratic equation $3x^2 - kx - 1 = 0$, where k is a positive integer, has roots α and β

(a) Show that $\alpha^2 + \beta^2 = \frac{k^2 + 6}{9}$ (3)

Given that $\alpha^4 + \beta^4 = \frac{466}{81}$

(b) find the value of k (5)

- (c) Hence form an equation, with integer coefficients, which has roots

$$\frac{\alpha^3 + \beta}{\beta} \text{ and } \frac{\beta^3 + \alpha}{\alpha}$$

(6)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total for Question 8 is 14 marks)

Question 9 continued

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 12 marks)

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10 (a) Show that $\frac{9^{3y}}{243} = 3^{(6y-5)}$

(4)

(b) Solve the simultaneous equations

$$\frac{9^{3y}}{243} = 27^{(x-2)}$$

$$\log_{10} \sqrt{6xy} = \log_4 2$$

(9)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 13 marks)

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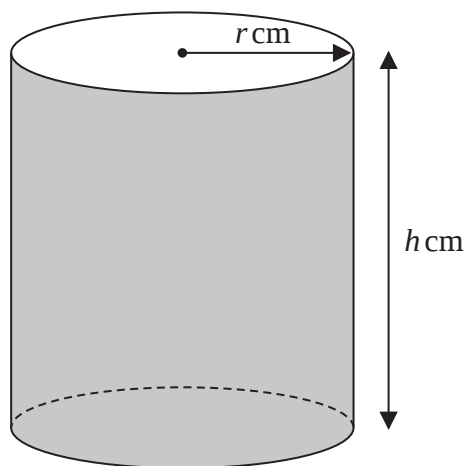
Diagram **NOT** accurately drawn**Figure 4**

Figure 4 shows an open container in the shape of a cylinder with radius r cm and height h cm.

Given that the total surface area of the container is $625\pi\text{ cm}^2$

(a) show that

$$h = \frac{625 - r^2}{2r} \quad (3)$$

The volume of the container is $V\text{ cm}^3$

Given that r can vary,

(b) use calculus to find the value, to 3 significant figures, of r for which V is a maximum.

Justify that this value of r gives a maximum value of V

(6)

(c) For the value of r found in part (b), find the corresponding value, to 3 significant figures, of h

(1)

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Question 11 continued

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(Total for Question 11 is 10 marks)**TOTAL FOR PAPER IS 100 MARKS**